GRADE 9 · PRE-AP FOUNDATIONS

AP Math Readiness Guide

What a 9th grader should know, practice, and be able to do to build toward AP Precalculus, AP Calculus, and AP Statistics.

"There is no royal road to geometry."

— Euclid, as told by Proclus (5th century)

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How to use this guide: sections marked CORE are the Algebra 1 skills most 9th-grade classes cover. Sections marked GET AHEAD are Geometry and Algebra 2 ideas usually taught in 10th or 11th grade — learning them early pays off in every AP math course.

Name: _____ Class / Period: _____

1 The Road to AP Math

AP math courses are college-level classes at the top of a four-year ladder. Most students take them in 11th or 12th grade, so 9th grade is where you build the algebra everything else stands on.

Grade 9

Algebra 1 · or Geometry if you took Algebra 1 in 8th grade

- Linear equations, inequalities, and systems
- Functions and their graphs
- Exponents, polynomials, and factoring
- Quadratics
- Exponential growth and decay
- Data displays and lines of fit

Grade 10

Geometry · or Algebra 2

- Proof and logical reasoning
- Congruence and similarity
- Right-triangle trigonometry
- Circles, area, and volume
- Coordinate geometry

Grade 11

Algebra 2 · or AP Precalculus

- Polynomial and rational functions
- Exponentials and logarithms
- Transformations of functions
- Sequences and series
- Trigonometric functions

Grade 12

AP Precalculus, AP Calculus AB/BC, or AP Statistics

- Full practice exams
- Calculus: limits, derivatives, and integrals
- Statistics: study design and inference
- Precalculus: modeling with functions

Schools sequence courses differently. To reach AP Calculus by 12th grade after Algebra 1 in 9th grade, students usually double up (for example, Geometry and Algebra 2 in the same year) or take a summer course — ask your counselor early. AP Statistics usually follows Algebra 2 and can be taken alongside another math course.

What AP math actually tests

AP exams do **not** just check whether you can follow a procedure. They measure three kinds of mathematical thinking. Fast, accurate algebra is the tool; **reasoning and explaining** are the goal.

Practice	What it means	Grade 9 version
Procedures	Carry out algebra, arithmetic, and calculator work accurately, and choose an efficient method.	Solve multi-step equations, factor quadratics, and use exponent rules without a calculator.
Connecting representations	Move between the four ways to show a relationship: **graphical** (a graph), **numerical** (a table), **analytical** (an equation), and **verbal** (words).	Given a line or an exponential model in any one form, produce the other three.
Justification and communication	Explain why an answer is right, with correct notation, units, and a sentence in context.	Write one complete sentence that says what a slope, intercept, or solution means in the problem.

Each course names these in its own words. AP Precalculus: procedural and symbolic fluency, multiple representations, and communication and reasoning. AP Calculus: implementing mathematical processes, connecting representations, justification, and communication and notation. AP Statistics: selecting statistical methods, data analysis, using probability and simulation, and statistical argumentation.

The habits that matter most in 9th grade

Show every step

Write one step per line, lined up at the equal sign. AP readers can only give credit for work they can see, and clean steps help you catch your own mistakes.

Check your answer

Substitute the solution back into the original equation, or ask, "Is this reasonable?" A negative length or a probability of 4 means something went wrong.

Mix your practice

Twenty problems of the same kind feel good but fade fast. Mix in a few older problems every week, so you learn to recognize which tool a problem needs.

Explain it in words

After a problem, write one sentence about what the answer means. "$x = 4$" becomes **"the two plans cost the same after 4 months."**

2 The AP Exams at a Glance

Every AP math exam is given in May and scored from 1 to 5. Many colleges award credit or placement for a 3 or higher. Knowing the formats now helps you practice the right skills for four years.

Section	Part	Questions	Time	Calculator	Share of score
AP Precalculus					
I. Multiple choice 40 questions · 2 hr	A	28	80 min	No calculator	**62.5%**
	B	12	40 min	Graphing calculator required	
II. Free response 4 questions · 1 hr	A	2	30 min	Graphing calculator	**37.5%**
	B	2	30 min	No calculator	
AP Calculus AB and AP Calculus BC (same format)					
I. Multiple choice 45 questions · 1 hr 45 min	A	30	60 min	No calculator	**50%**
	B	15	45 min	Graphing calculator required	
II. Free response 6 questions · 1 hr 30 min	A	2	30 min	Graphing calculator	**50%**
	B	4	60 min	No calculator	

Section	Part	Questions	Time	Calculator	Share of score
AP Statistics					
I. Multiple choice 40 questions · 1 hr 30 min	—	40	1 hr 30 min	Graphing calculator	**50%**
II. Free response 6 questions · 1 hr 30 min	Q1–5	5 short	1 hr 30 min	Graphing calculator	**50%**
	Q6	1 long			

Exam	What it covers
AP Precalculus	Units on the exam: Polynomial and Rational Functions · Exponential and Logarithmic Functions · Trigonometric and Polar Functions. A fourth unit, Functions Involving Parameters, Vectors, and Matrices, is taught in the course but is not on the exam.
AP Calculus AB	Limits, derivatives, integrals, and their applications.
AP Calculus BC	Everything in AB, plus parametric, polar, and vector functions and infinite series.
AP Statistics	Exploring data, collecting data (sampling and experiments), probability, and statistical inference. Free-response question 6 is a longer investigative task.

Formats per the College Board's Course and Exam Descriptions. Details can change: check AP Central (apcentral.collegeboard.org) during the year you take the exam.

How free-response answers are scored

Trained readers score each free-response question with a point-by-point rubric. Points go to answers that:

- **Show the work** — the AP Calculus directions warn that answers without supporting work usually do not receive credit.
- **Use correct notation** — $f(3) = 7$, and equal signs only between things that really are equal.
- **Answer in context** — with units and a sentence about the real situation.
- **Justify conclusions** — name the reason (a rate, a sign change, a comparison), not just the result.
- **Use the calculator wisely** — write the setup, then the result. AP Calculus wants decimal answers correct to three places unless a question says otherwise.
- **Attempt every part** — each part earns its own points, so never leave one blank.

> **The 9th-grade takeaway**
>
> You do not need to be a "math person" to do well in AP math. You need fluent algebra, the habit of showing work, and the ability to say what an answer means. Practice all three from day one.

3 Numbers and Expressions

Every AP problem ends in arithmetic and algebra. When these moves are automatic, your brain is free for the actual problem.

Order of operations CORE

Work **grouping symbols** first (parentheses, brackets, fraction bars, absolute value), then **exponents**, then **multiplication and division** from left to right, then **addition and subtraction** from left to right.

$8 - 2(3-5)^2 \div 4$	start
$= 8 - 2(-2)^2 \div 4$	parentheses first
$= 8 - 2(4) \div 4$	exponent
$= 8 - 8 \div 4$	multiply
$= 8 - 2 = 6$	divide, then subtract

Watch the negative sign

$-3^2 = -9$, because the exponent applies only to the 3. $(-3)^2 = 9$. On a calculator, type the parentheses.

Fractions and signed numbers CORE

Operation	Rule	Example
Add or subtract	Rewrite with a common denominator, then add the numerators.	$\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$
Multiply	Numerator times numerator, denominator times denominator; simplify.	$\frac{2}{3} \cdot \frac{9}{10} = \frac{18}{30} = \frac{3}{5}$
Divide	Multiply by the reciprocal of the second fraction.	$\frac{2}{3} \div \frac{4}{9} = \frac{2}{3} \cdot \frac{9}{4} = \frac{3}{2}$
Signs	Same signs multiply to a positive; different signs to a negative. Subtracting a negative adds.	$(-4)(-3) = 12 \cdot (-4)(3) = -12 \cdot 5 - (-3) = 8$

Exponent rules CORE

Rule	In symbols	Example
Product	$a^m \cdot a^n = a^{m+n}$	$x^2 \cdot x^5 = x^7$
Quotient	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^9}{x^3} = x^6$
Power of a power	$(a^m)^n = a^{mn}$	$(x^3)^4 = x^{12}$
Power of a product or quotient	$(ab)^n = a^n b^n \cdot (\frac{a}{b})^n = \frac{a^n}{b^n}$	$(2x)^3 = 8x^3 \cdot (\frac{x}{3})^2 = \frac{x^2}{9}$

Rule	In symbols	Example
Zero exponent	$a^0 = 1$ (for $a \neq 0$)	$7^0 = 1 \cdot (3xy)^0 = 1$
Negative exponent	$a^{-n} = \frac{1}{a^n}$ — a reciprocal, not a negative number	$2^{-3} = \frac{1}{8} \cdot x^{-2} = \frac{1}{x^2}$
Rational (fractional) exponents GET AHEAD		
Unit fraction	$a^{1/n} = \sqrt[n]{a}$	$25^{1/2} = 5 \cdot 8^{1/3} = \sqrt[3]{8} = 2$
Any fraction	$a^{m/n} = (\sqrt[n]{a})^m$	$27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9$

Simplifying square roots CORE

Pull out the largest perfect-square factor: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$\sqrt{50} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$
$\sqrt{48} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$

Know the perfect squares: 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144.

> **Get ahead: rationalize the denominator**
>
> Multiply top and bottom by the root: $\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$.

Scientific notation CORE

Write a number as $a \times 10^n$ with $1 \leq a < 10$.

$52{,}000 = 5.2 \times 10^4$
$0.00052 = 5.2 \times 10^{-4}$
$(3 \times 10^4)(5 \times 10^6) = 15 \times 10^{10} = 1.5 \times 10^{11}$

A calculator may show 1.5E11 — that means 1.5×10^{11}.

Properties and like terms CORE

Distributive: $a(b + c) = ab + ac$, so $-3(x - 4) = -3x + 12$.

Like terms have the same variable to the same power: $4x^2 + 3x - x^2 + 5x - 2 = 3x^2 + 8x - 2$. x^2 and x are *not* like terms.

Types of numbers			*each set contains the ones above it*
Natural numbers	1, 2, 3, …	**Rational numbers**	any fraction of integers: $\frac{3}{4}$, -2, 0.5, 0.333…
Whole numbers	0, 1, 2, 3, …	**Irrational numbers**	cannot be written as a fraction: $\sqrt{2}$, π
Integers	…, −2, −1, 0, 1, 2, …	**Real numbers**	all rational and irrational numbers

4 Equations and Inequalities

Solving is undoing: whatever you do to one side, you do to the other, until the variable stands alone.

Multi-step linear equations CORE

Distribute, combine like terms on each side, collect the variable on one side, then divide. Check by substituting into the **original** equation.

$5(x-2)-3x=4x+6$	start
$5x-10-3x=4x+6$	distribute
$2x-10=4x+6$	combine like terms
$-2x-10=6$	subtract $4x$
$-2x=16$	add 10
$x=-8$	divide by -2

Check: $5(-10)-3(-8)=-26$ and $4(-8)+6=-26$ ✓

Fractions? Clear them first

Multiply every term by the least common denominator.

$\frac{x}{3}+\frac{x}{4}=7$ → multiply by 12 → $4x+3x=84$, so $x=12$.

Literal equations CORE

Solve for one letter in terms of the others — the same moves, with letters.

$P=2l+2w \rightarrow w=\frac{P-2l}{2}$

$A=\frac{1}{2}bh \rightarrow h=\frac{2A}{b}$

Inequalities CORE

Solve like an equation, with one extra rule: **when you multiply or divide both sides by a negative number, flip the inequality sign.** On a number line, an open circle means the endpoint is not included (<, >); a closed circle means it is (≤, ≥).

$-2x+5>11$	start
$-2x>6$	subtract 5
$x<-3$	divide by -2: flip!



$-1\le 2x+3<9$	compound inequality
$-4\le 2x<6$	subtract 3 from all parts
$-2\le x<3$	divide all parts by 2



GET AHEAD Interval notation: $x<-3$ is $(-\infty,-3)$ and $-2\le x<3$ is $[-2,3)$. A square bracket means the endpoint is included.

Absolute value equations CORE

$|A| = k$ means A is k units from 0, so $A = k$ or $A = -k$.

$|x - 4| = 7 \rightarrow x - 4 = 7$ or $x - 4 = -7 \rightarrow x = 11$ or $x = -3$.

Isolate the absolute value first: $2|x + 1| - 3 = 7 \rightarrow |x + 1| = 5 \rightarrow x = 4$ or $x = -6$. If it equals a negative number, there is no solution.

> **Get ahead: absolute value inequalities**
>
> "Less than" means **between**: $|x - 1| < 3 \rightarrow -2 < x < 4$.
>
> "Greater than" means **outside**: $|x| > 5 \rightarrow x < -5$ or $x > 5$.

Systems of equations CORE

Method	Best when	Example
Graphing	You want to see the solution, or an estimate is enough.	The solution is the point where the two lines cross.
Substitution	One variable is already alone.	$y = 2x + 1$ and $3x + y = 16$: $3x + (2x + 1) = 16 \rightarrow x = 3, y = 7$. Solution **(3, 7)**.
Elimination	Coefficients match or are easy to match.	$3x + 2y = 12$ and $5x - 2y = 4$: add $\rightarrow 8x = 16 \rightarrow x = 2, y = 3$. Solution **(2, 3)**.

One solution, none, or infinitely many? CORE

When you solve, you end with...	It means	The graphs
one value, like $x = 3$	**exactly one solution**	lines cross once
a false statement, like $6 = 5$	**no solution**	parallel lines (same slope, different intercepts)
a true statement, like $6 = 6$	**infinitely many solutions**	the same line

Example: $2(x + 3) = 2x + 5$ becomes $6 = 5$ (no solution); $2(x + 3) = 2x + 6$ becomes $6 = 6$ (every x works).

Translating words into algebra			*"less than" flips the order*
5 more than a number	$x + 5$	**5 less than a number**	$x - 5$ (not $5 - x$)
twice a number	$2x$	**the quotient of a number and 4**	$\frac{x}{4}$
3 times the sum of a number and 2	$3(x + 2)$	**6 times a number, minus 1**	$6x - 1$
a number is at least 18	$x \geq 18$	**a number is at most 18**	$x \leq 18$

5 Functions

Functions are the language of AP Precalculus and AP Calculus. Every course after Algebra 1 is, in some way, a study of functions.

What is a function? CORE

A **function** assigns each input **exactly one** output. Inputs are usually x; outputs are y or $f(x)$.

Function notation

$f(x) = x^2 - 2x$ is read "f of x." $f(-1)$ is the output when $x = -1$: $f(-1) = (-1)^2 - 2(-1) = 3$. So the point **(−1, 3)** is on the graph. $f(x)$ does **not** mean f times x.

The vertical line test

A graph shows a function if no vertical line touches it more than once. A parabola that opens up passes; a circle fails, because one x gives two y-values.

Domain and range

The **domain** is every input allowed; the **range** is every output that actually happens. In Algebra 1, two things shrink a domain: **dividing by zero** and **the square root of a negative**.

Function	Domain	Range
$f(x) = 3x - 1$	all real numbers	all real numbers
$g(x) = \dfrac{1}{x - 2}$	$x \neq 2$ (cannot divide by 0)	$y \neq 0$
$h(x) = \sqrt{x + 3}$	$x \geq -3$ (inside the root must be ≥ 0)	$y \geq 0$
$k(x) = x^2 + 1$	all real numbers	$y \geq 1$

The parent functions CORE

Learn these six shapes by heart. Almost every function in AP Precalculus is one of them, moved or stretched.

Function	Equation	Graph	Key features	Domain	Range
Linear	$f(x) = x$		straight line; constant rate of change (slope 1)	all reals	all reals
Quadratic	$f(x) = x^2$		U-shaped parabola; vertex (0, 0); symmetric about the y-axis	all reals	$y \geq 0$
Absolute value	$f(x) = \lvert x \rvert$		V shape; vertex (0, 0); slopes −1 and 1	all reals	$y \geq 0$
Square root	$f(x) = \sqrt{x}$		starts at (0, 0); rises more and more slowly	$x \geq 0$	$y \geq 0$

Function	Equation	Graph	Key features	Domain	Range
Cubic	$f(x) = x^3$		S-shaped curve through (0, 0); flattens out at the origin	all reals	all reals
Exponential	$f(x) = 2^x$		y-intercept (0, 1); grows faster and faster; approaches $y = 0$ on the left (a horizontal asymptote)	all reals	$y > 0$

Transformations GET AHEAD

Every function in the family $y = a \cdot f(x - h) + k$ is the parent graph moved, stretched, or flipped.

Change	Effect on the graph	Example with $y = x^2$
$f(x) + k$	shift up k (down if $k < 0$)	$y = x^2 + 3$: up 3
$f(x - h)$	shift right h — the **opposite** of the sign you see inside	$y = (x - 4)^2$: right 4; $y = (x + 4)^2$: left 4
$-f(x)$	reflect over the x-axis	$y = -x^2$: opens down
$a \cdot f(x)$	vertical stretch if $a > 1$ (narrower); compression if $0 < a < 1$ (wider)	$y = 3x^2$: narrower; $y = \frac{1}{2}x^2$: wider
$f(-x)$	reflect over the y-axis	$y = 2^{-x} = (\frac{1}{2})^x$

Put it together: $y = -2(x + 3)^2 + 1$ is $y = x^2$ moved left 3, stretched by 2, flipped to open down, and moved up 1. Its vertex is $(-3, 1)$.

Average rate of change CORE

$$\text{average rate of change} = \frac{f(b) - f(a)}{b - a}$$

It is the change in output divided by the change in input — the slope of the line through two points on the graph.

For $f(x) = x^2$ from $x = 1$ to $x = 3$: $\frac{9 - 1}{3 - 1} = 4$. On average, the output rises 4 units for each unit of input on that interval.

Where this leads: calculus

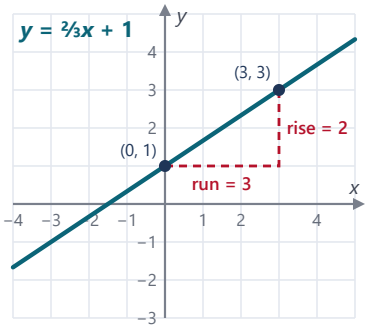
In AP Calculus you shrink the interval until the two points nearly touch. The average rate of change becomes the **instantaneous** rate of change — the derivative. If you understand the slope between two points, you already understand the starting idea of calculus.

Reading a graph or table			*words AP questions use constantly*
zero (root)	an input where $f(x) = 0$; an x-intercept	**y-intercept**	the output at $x = 0$, which is $f(0)$
increasing	outputs go up as x moves right	**decreasing**	outputs go down as x moves right
maximum	the highest output (a peak)	**minimum**	the lowest output (a valley)
positive	where the graph is above the x-axis	**negative**	where the graph is below the x-axis

6 Linear Functions

A linear function changes by the same amount every step. That constant rate of change — the slope — is the most important number in Algebra 1.

Slope CORE



Slope = rise ÷ run = 2 ÷ 3.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Slope is a **rate**: how much y changes for each 1-unit increase in x. Its units are output units per input unit — dollars per hour, miles per gallon, degrees per minute.

Through (0, 1) and (3, 3): $m = \frac{3 - 1}{3 - 0} = \frac{2}{3}$.

- **Positive** slope: rises left to right. **Negative**: falls.
- **Zero** slope: horizontal line, $y = c$.
- **Undefined** slope: vertical line, $x = c$.

Three forms of a line CORE

Form	Equation	Best for	Example
Slope-intercept	$y = mx + b$	graphing; reading the rate (m) and the starting value (b)	$y = 2x - 3$
Point-slope	$y - y_1 = m(x - x_1)$	writing a line from one point and the slope	$y - 1 = 2(x - 2)$
Standard	$Ax + By = C$	finding both intercepts quickly	$2x - y = 3$

All three examples are the same line: $y - 1 = 2(x - 2)$ simplifies to $y = 2x - 3$, and so does $2x - y = 3$.

Writing a line from two points

Points $(-1, 7)$ and $(3, -1)$

$$m = \frac{-1-7}{3-(-1)} = \frac{-8}{4} = -2 \quad \text{slope}$$

$$y - 7 = -2(x+1) \quad \text{point-slope}$$

$$y = -2x + 5 \quad \text{simplify}$$

Check $(3, -1)$: $-2(3) + 5 = -1$ ✓

Parallel and perpendicular

Parallel lines have equal slopes: $y = 3x + 1$ and $y = 3x - 4$. **Perpendicular** lines have opposite reciprocal slopes, whose product is -1: $\frac{2}{3}$ and $-\frac{3}{2}$.

Intercepts from standard form

In $2x + 5y = 10$, set $y = 0$ for the x-intercept $(5, 0)$; set $x = 0$ for the y-intercept $(0, 2)$.

Linear models and units CORE

A taxi charges \$3.50 plus \$2.25 per mile. The cost of an m-mile ride is $C(m) = 2.25m + 3.50$.

Part	What it means in context
Slope = 2.25	The cost increases by \$2.25 for each additional mile.
Vertical intercept = 3.50	The ride costs \$3.50 before the taxi moves (0 miles).
$C(8) = 21.50$	An 8-mile ride costs \$21.50.
$C(m) = 30 \rightarrow m \approx 11.8$	With \$30 you can ride about 11.8 miles.

Always say the units

"The slope is 2.25" is an incomplete answer on an AP free-response question. "The cost increases by \$2.25 per mile" is complete: it names the quantity, the rate, and the units.

Arithmetic sequences CORE

$$a_n = a_1 + (n-1)d$$

An arithmetic sequence adds the same amount d (the common difference) at every step, so it is a linear function whose inputs are 1, 2, 3, …

For 5, 8, 11, 14, …: $d = 3$ and $a_n = 5 + 3(n-1) = 3n + 2$, so the 20th term is $a_{20} = 62$.

Where this leads: AP Precalculus

AP Precalculus compares **arithmetic** sequences (add d each step) with **geometric** sequences (multiply by r each step, like 2, 6, 18, 54, …). They are the step-by-step versions of linear and exponential functions — see section 8.

7 Polynomials, Factoring, and Quadratics

Quadratics are the first curved functions you study in depth. Factoring is the skill that unlocks them — and it comes back in every later course.

Polynomial basics CORE

A polynomial is a sum of terms like $3x^2$, $-5x$, and 7. Write it in **standard form**, highest power first. The **degree** is the highest power; the **leading coefficient** is the number in front of it. $4x^3 - x + 9$ has degree 3 and leading coefficient 4.

Add or subtract: distribute the minus, then combine like terms

$$(3x^2 - 2x + 5) - (x^2 + 4x - 1)$$
$$= 3x^2 - 2x + 5 - x^2 - 4x + 1$$
$$= 2x^2 - 6x + 6$$

Multiply: every term times every term

$$(2x + 3)(x - 4)$$
$$= 2x^2 - 8x + 3x - 12$$
$$= 2x^2 - 5x - 12$$

Special products CORE

Pattern	Expansion	Example
$(a + b)^2$	$a^2 + 2ab + b^2$	$(x + 5)^2 = x^2 + 10x + 25$
$(a - b)^2$	$a^2 - 2ab + b^2$	$(2x - 3)^2 = 4x^2 - 12x + 9$
$(a + b)(a - b)$	$a^2 - b^2$	$(x + 7)(x - 7) = x^2 - 49$

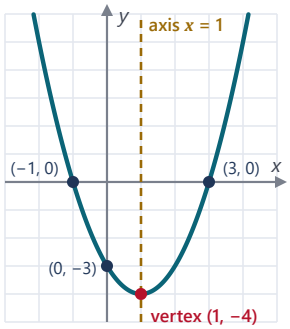
Factoring: a decision guide CORE

Factoring undoes multiplying. Work down this list in order, every time, and check by multiplying back.

Look for	Method	Example
1. A factor common to every term	Factor out the greatest common factor (GCF) — always first	$4x^3 - 8x^2 = 4x^2(x - 2)$
2. Two terms: a square minus a square	Difference of squares: $a^2 - b^2 = (a + b)(a - b)$	$4x^2 - 25 = (2x + 5)(2x - 5)$
3. Three terms: $x^2 + bx + c$	Two numbers that multiply to c and add to b	$x^2 - x - 12 = (x - 4)(x + 3)$
4. Three terms: $ax^2 + bx + c$**,** $a \neq 1$	AC method: two numbers that multiply to ac and add to b; split the middle term; group	$3x^2 + 10x + 8$: $ac = 24 \rightarrow 6$ and 4 $= 3x(x + 2) + 4(x + 2) = (3x + 4)(x + 2)$
5. Four terms	Grouping: factor each pair, then the common binomial	$x^3 + 3x^2 + 2x + 6 = x^2(x + 3) + 2(x + 3)$ $= (x^2 + 2)(x + 3)$

A perfect-square trinomial such as $x^2 + 6x + 9$ factors as $(x + 3)^2$. A sum of squares such as $x^2 + 9$ does not factor over the real numbers.

Three forms of a quadratic CORE



One parabola, three equations.

Form	Equation	Shows you
Standard	$y = ax^2 + bx + c$ $y = x^2 - 2x - 3$	the *y*-intercept, (0, *c*)
Vertex	$y = a(x - h)^2 + k$ $y = (x - 1)^2 - 4$	the vertex, (*h*, *k*)
Factored	$y = a(x - r)(x - s)$ $y = (x - 3)(x + 1)$	the zeros, *r* and *s*

If $a > 0$, the parabola opens up and the vertex is a minimum. If $a < 0$, it opens down and the vertex is a maximum. The axis of symmetry passes through the vertex, halfway between the zeros.

Four ways to solve a quadratic equation

First get 0 on one side. The **zero-product property** says that if $AB = 0$, then $A = 0$ or $B = 0$ — that is why factoring solves equations.

Method	Use it when	Example
Factoring CORE	the quadratic factors nicely	$x^2 + x - 6 = 0 \to (x + 3)(x - 2) = 0 \to x = -3$ or $x = 2$
Square roots CORE	there is no *x*-term, or a square is already isolated	$(x - 2)^2 = 9 \to x - 2 = \pm 3 \to x = 5$ or $x = -1$
Completing the square GET AHEAD	$a = 1$ and b is even; you want vertex form	$x^2 + 6x - 7 = 0 \to x^2 + 6x + 9 = 16 \to (x + 3)^2 = 16 \to x = 1$ or $x = -7$
Quadratic formula CORE	always works	$2x^2 - 4x - 1 = 0 \to x = \frac{4 \pm \sqrt{24}}{4} = \frac{2 \pm \sqrt{6}}{2} \approx 2.22$ or -0.22

The quadratic formula, for $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac$	Real solutions	Graph
positive	two	crosses the *x*-axis twice
zero	one	touches at the vertex
negative	none*	never touches

* Two complex solutions — a Get Ahead idea from Algebra 2.

Finding the vertex from standard form

The axis of symmetry is $x = -\frac{b}{2a}$. Substitute that x to get the y-value.

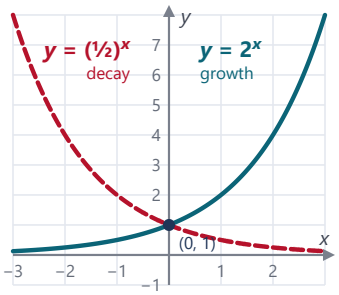
For $y = 2x^2 - 8x + 3$: $x = \frac{8}{4} = 2$, and $y = 2(4) - 16 + 3 = -5$.

Vertex $(2, -5)$; axis of symmetry $x = 2$; minimum value -5, because $a > 0$.

8 Exponential Functions

Linear functions add the same amount each step. Exponential functions multiply by the same factor. That one difference explains compound interest, population growth, and viral videos.

The exponential model CORE



Growth (solid) and decay (dashed) through (0, 1).

$$y = a \cdot b^x$$

- ***a*** is the **initial value**: the output when $x = 0$ (the y-intercept).
- ***b*** is the **growth or decay factor**: multiply by b each step.
- $b > 1$ means growth; $0 < b < 1$ means decay.
- The graph never touches the x-axis: $y = 0$ is a horizontal asymptote.

Percent change CORE

Description	Factor b
grows 5% per year	$1 + 0.05 = 1.05$
loses 20% per year	$1 - 0.20 = 0.80$
doubles every period	2
halves every period (half-life)	$½ = 0.5$

Growth: a town of 2,500 people grows 4% per year

$P(t) = 2500(1.04)^t$

$P(10) \approx 3{,}701$ people

Decay: an $800 phone loses 25% of its value each year

$V(t) = 800(0.75)^t$

$V(3) = \$337.50$

Half-life: 80 mg of a medicine with a half-life of 6 hours leaves $80(½)^{t/6}$ mg after t hours. After 18 hours (three half-lives): $80(½)^3 = 10$ mg.

Compound interest CORE

$$A = P(1 + \frac{r}{n})^{nt}$$

P = starting amount (principal) · r = annual rate as a decimal · n = times compounded per year · t = years

$2,000 at 6% compounded quarterly for 5 years:

$A = 2000(1 + \frac{0.06}{4})^{4 \cdot 5} = 2000(1.015)^{20} \approx$ $2,693.71

Get ahead: the number *e*

When interest is compounded continuously, $A = Pe^{rt}$, where $e \approx 2.718$. The number e shows up all over AP Precalculus and AP Calculus.

Linear or exponential? CORE

	Linear	Exponential
Each step	adds the same amount	multiplies by the same factor
In a table	equal differences	equal ratios
Graph	straight line	curve that bends upward (growth) or levels off toward 0 (decay)
Equation	$y = mx + b$	$y = a \cdot b^x$
Outputs for x = 0 to 4	2, 5, 8, 11, 14 (add 3 each time)	2, 6, 18, 54, 162 (multiply by 3 each time)

Exponential growth always wins

An increasing exponential function eventually passes any increasing linear function, no matter how big the linear head start. Practice question I asks you to show it with numbers.

Get ahead: logarithms

A logarithm is an exponent. $\log_b x = y$ means $b^y = x$. So $\log_2 8 = 3$ and $\log 1000 = 3$ (base 10). Logs solve for an exponent: $2^x = 50 \rightarrow x = \frac{\log 50}{\log 2} \approx 5.64$.

9 Geometry and Trigonometry Essentials

GET AHEAD

Most of this is taught in Geometry. AP Precalculus and AP Calculus assume you know it cold — especially right triangles and the first ideas of trigonometry.

The Pythagorean theorem

$a^2 + b^2 = c^2$ *c* is the hypotenuse, opposite the right angle

Legs 6 and 8: $c^2 = 36 + 64 = 100$, so $c = 10$.

Common right triangles (Pythagorean triples): 3-4-5, 5-12-13, 8-15-17, 7-24-25, and their multiples, like 6-8-10.

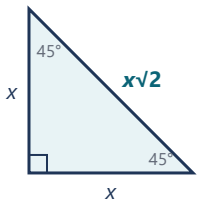
Distance and midpoint

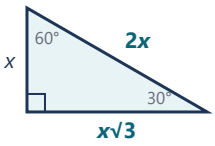
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

From (1, 2) to (7, 10): $d = \sqrt{6^2 + 8^2} = 10$; midpoint (4, 6). The distance formula is the Pythagorean theorem on a grid.

Special right triangles





45°-45°-90°: the legs are equal; hypotenuse = leg × √2. A square with side 5 has a diagonal of $5\sqrt{2} \approx 7.07$.

30°-60°-90°: hypotenuse = 2 × short leg; long leg = short leg × √3. The short leg is opposite the 30° angle. An equilateral triangle with side 10 has a height of $5\sqrt{3} \approx 8.66$.

Right-triangle trigonometry: SOH-CAH-TOA

Ratio	Definition	Memory
$\sin\theta$	opposite ÷ hypotenuse	**SOH**
$\cos\theta$	adjacent ÷ hypotenuse	**CAH**
$\tan\theta$	opposite ÷ adjacent	**TOA**

Find a side. Hypotenuse 20, angle 35°: opposite $= 20\sin 35° \approx 11.47$.

Find an angle. Opposite 5, adjacent 12: $\tan\theta = \frac{5}{12}$, so $\theta = \tan^{-1}(\frac{5}{12}) \approx 22.6°$.

> **Degree mode or radian mode?**
>
> $\sin 30° = 0.5$. If your calculator says -0.988, it is in radian mode. Check the mode before every test — it is one of the most common lost points in AP Precalculus and AP Calculus.

Area and volume formulas			*B = area of the base*
Rectangle	$A = lw$	Rectangular prism	$V = lwh$
Parallelogram	$A = bh$	Cylinder	$V = \pi r^2 h$
Triangle	$A = \frac{1}{2}bh$	Pyramid	$V = \frac{1}{3}Bh$
Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$	Cone	$V = \frac{1}{3}\pi r^2 h$
Circle	$A = \pi r^2 \cdot C = 2\pi r$	Sphere	$V = \frac{4}{3}\pi r^3$

A first look at radians and the unit circle

Calculus measures angles in **radians**. A full turn is $360° = 2\pi$ radians, so $180° = \pi$ radians. To convert degrees to radians, multiply by $\frac{\pi}{180}$. On the **unit circle** (radius 1, centered at the origin), the point at angle θ is $(\cos\theta, \sin\theta)$. That is why $\sin^2\theta + \cos^2\theta = 1$ — the Pythagorean theorem again.

Degrees	0°	30°	45°	60°	90°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

10 Data and Statistics

AP Statistics is about reasoning with data: describing it, collecting it well, and deciding what it lets you conclude. The vocabulary starts now.

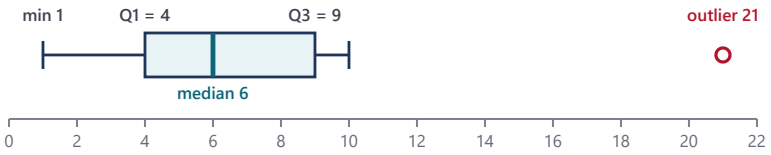
Center and spread CORE

Measure	How to find it	Use it when
Center		
Mean	add the values; divide by how many there are	the data is roughly symmetric, with no outliers
Median	the middle of the sorted list (average the two middle values if the count is even)	the data is skewed or has outliers
Mode	the most frequent value	you want the most common value or category
Spread		
Range	maximum − minimum	a quick look (one outlier can distort it)
IQR	$Q3 - Q1$: the spread of the middle 50% of the data	you are using the median
Standard deviation	roughly the typical distance of the values from the mean (your calculator finds it)	you are using the mean

Five-number summary, outliers, and box plots CORE

Books read in a year by 11 students: **1, 3, 4, 4, 5, 6, 7, 8, 9, 10, 21**

- **Median:** the 6th of 11 values = **6**.
- **Q1** = median of the lower half (1, 3, 4, 4, 5) = **4**. **Q3** = median of the upper half (7, 8, 9, 10, 21) = **9**. With an odd count, leave the median out of both halves.
- **IQR** = 9 − 4 = **5**.
- **Outlier rule:** a value is an outlier if it is below $Q1 - 1.5 \times IQR = -3.5$ or above $Q3 + 1.5 \times IQR = 16.5$. So **21 is an outlier**.
- **Mean** = 78 ÷ 11 ≈ 7.09 — pulled above the median by the outlier.



Five-number summary: 1, 4, 6, 9, 21. The whisker stops at 10, the largest value that is not an outlier; 21 is plotted as a dot.

Shape: describe it every time

Describe a distribution with **S**hape, **O**utliers, **C**enter, and **S**pread (SOCS) — always in context.

- **Symmetric:** mean ≈ median.
- **Skewed right:** a long tail to the right; the mean is usually greater than the median.
- **Skewed left:** a long tail to the left; the mean is usually less than the median.

Which display?

Dot plot	small data sets; every value visible
Histogram	larger data sets; shows shape (bars are intervals)
Box plot	comparing groups; spotting outliers
Two-way table	two categorical variables
Scatterplot	two quantitative variables

Two-way tables CORE

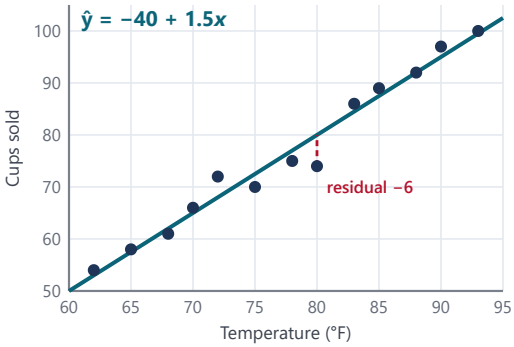
200 students surveyed	Prefer digital	Prefer print	Total
9th grade	50	30	80
10th grade	60	60	120
Total	110	90	200

Overall, $110 \div 200 = 55\%$ prefer digital.

Of the 9th graders, $50 \div 80 = 62.5\%$ prefer digital; of the 10th graders, $60 \div 120 = 50\%$. In this sample, 9th graders were more likely to prefer digital.

Compare percentages within each group, not raw counts: there were more 10th graders.

Scatterplots, correlation, and lines of fit CORE



A lemonade stand: temperature and cups sold on 13 days.

Describe a scatterplot by **direction** (positive or negative), **form** (linear or curved), **strength**, and any unusual points.

The correlation ***r*** runs from −1 to 1. Close to ±1 means a strong linear pattern; close to 0 means a weak one. Here $r \approx 0.99$: strong, positive, and linear.

The line of best fit is $\hat{y} = -40 + 1.5x$, where x is the temperature (°F) and $\hat{y}$ is the predicted number of cups sold.

Part	What it means in context
Slope 1.5	For each additional 1°F, the predicted number of cups sold increases by 1.5.
Intercept −40	At 0°F the model predicts −40 cups — impossible, because 0°F is far outside the data. Predicting outside the data is called **extrapolation**.
Prediction	At 80°F: $\hat{y} = -40 + 1.5(80) = 80$ cups.
Residual	actual − predicted. On the 80°F day the stand sold 74 cups: $74 - 80 = -6$, so the model overestimated by 6 cups.

Correlation is not causation

Ice cream sales and sunburns rise together, but ice cream does not cause sunburns: sunny weather drives both. Only a well-designed experiment can show cause and effect — a big idea in AP Statistics.

Basic probability CORE

Rule	Formula	Example
Probability	$P(A) = \frac{\text{favorable outcomes}}{\text{total outcomes}}$ (equally likely outcomes)	An even number on a die: $\frac{3}{6} = \frac{1}{2}$
Complement	$P(\text{not } A) = 1 - P(A)$	Not rolling a 6: $1 - \frac{1}{6} = \frac{5}{6}$
"And" (independent events)	$P(A \text{ and } B) = P(A) \cdot P(B)$	Two heads in two coin flips: ½ · ½ = ¼
"Or" GET AHEAD	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$	A heart or a king from a deck: $\frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{4}{13}$

Every probability is between 0 (impossible) and 1 (certain).

11 Mistakes to Avoid

Most lost points come from a short list of algebra slips. Learn to spot these now, and they will never cost you on an AP exam.

Not this	This	Why
~~$(x+3)^2 = x^2 + 9$~~	$(x+3)^2 = x^2 + 6x + 9$	Squaring a sum means multiplying $(x+3)(x+3)$; keep the middle term.
~~$\sqrt{x^2+16} = x+4$~~	$\sqrt{x^2+16}$ **stays as it is**	Roots do not split over addition: $\sqrt{9+16} = 5$, not 3 + 4 = 7.
~~$-(x-5) = -x-5$~~	$-(x-5) = -x+5$	The negative multiplies every term inside.
~~$x^2 = 25$, so $x = 5$~~	$x = \pm 5$	Both 5^2 and $(-5)^2$ equal 25.
~~$-3x > 12$, so $x > -4$~~	$x < -4$	Dividing by a negative flips the inequality.
~~$x^2 = 4x$, so $x = 4$~~	$x(x-4) = 0$, **so** $x = 0$ **or** 4	Dividing both sides by x throws away the solution $x = 0$. Factor instead.
~~slope of (1, 2), (4, 8): $\frac{8-2}{1-4}$~~	$\frac{8-2}{4-1} = 2$	Subtract in the same order on top and bottom.

Not this	This	Why
$\frac{x+6}{6} = x$	$\frac{x+6}{6} = \frac{x}{6} + 1$	Cancel factors, not terms. $\frac{2x+6}{2} = x+3$ works because 2 divides every term.
$-3^2 = 9$	$-3^2 = -9$	The exponent applies only to the 3; $(-3)^2 = 9$.
$2^{-3} = -8$	$2^{-3} = \frac{1}{8}$	A negative exponent means a reciprocal, not a negative number.
$(2x)^3 = 2x^3$	$(2x)^3 = 8x^3$	The exponent applies to every factor inside the parentheses.
$\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$	$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$	Add fractions with a common denominator, never top plus top over bottom plus bottom.
$\sin 30° = -0.988$	$\sin 30° = 0.5$	The calculator was in radian mode. Check DEG or RAD.
"The slope is 4.5."	**"The predicted score rises 4.5 points per extra hour of study."**	AP answers need units and context.
"They are correlated, so one causes the other."	**"They are associated; an experiment is needed to show cause."**	Correlation is not causation.

Words that mean something different in math

These look like everyday words but have exact meanings on a math exam.

Word	Everyday meaning	In math
range	a variety; a stretch of mountains	for a function, the set of all outputs; in statistics, maximum − minimum
product	something you buy	the result of multiplying
mean	unkind	the average: sum ÷ count
root	part of a plant	a solution of $f(x) = 0$ (also called a zero or x-intercept)
degree	a diploma; a temperature reading	a unit of angle measure; the highest power in a polynomial
or	one or the other, not both	in probability, one or the other **or both**
random	weird, unexpected	chosen by chance, with known probabilities (AP Statistics)
significant	important	unlikely to happen by chance alone (AP Statistics)
expression	a look on your face	math with no equal sign — you simplify it; an **equation** has one — you solve it

12 Study Plan and Resources

Twenty focused minutes a day beats a three-hour cram the night before a test. Here is a routine that fits around other classes and activities.

When	What to do	Time
Every day	Do homework with every step shown · redo one problem you missed · two minutes of mental math (percents, fractions, squares)	20–30 min
Every week	A short mixed review of older topics · explain one problem out loud or in writing, as if you were teaching it · review your mistakes notebook	About 1 hr total
Every month	A timed mini-test without notes · graph three functions from class in Desmos and describe what each number in the equation does	30–45 min
Summer	A few Khan Academy lessons a week from next year's course · one challenge problem a week · redo the practice in section 14	A little, often

Free and widely used resources

Learn and practice

- **Khan Academy** (khanacademy.org) — free lessons and practice for Algebra 1, Geometry, Algebra 2, Precalculus, Calculus, and Statistics.
- **Desmos** (desmos.com) — a free online graphing calculator. Type any equation and see its graph instantly.
- **Your textbook** — the practice sets, chapter reviews, and answers to odd-numbered problems match your class exactly.
- **Your teacher** — office hours and tutoring are underused. Bring one specific question.

Challenge and the exam

- **AP Central** (apcentral.collegeboard.org) — the official Course and Exam Descriptions, scoring guidelines, and past free-response questions.
- **AP Classroom** — practice your teacher assigns once you are in an AP class.
- **Art of Problem Solving** (artofproblemsolving.com) — challenging problems, books, and a free online community for students who love math.
- **Math contests** — the AMC 10, from the Mathematical Association of America, rewards creative problem solving.

Make math part of your life, not just homework

- Estimate the tip or the sale price before the register does.
- Track a stat for your favorite team or game, and compute its mean and spread.
- Compare two phone or streaming plans with equations, and find where they cross.
- Graph your savings over a few months: is the pattern linear or exponential?
- Play logic puzzles like Sudoku or KenKen to build persistence.

13 End-of-Year Self-Check

Check off each statement when it is true for you. Anything unchecked in June is your summer goal.

Expressions and Equations

- [ ] I can simplify expressions with the order of operations, including negatives and exponents.
- [ ] I know the exponent rules and can use them without a calculator.
- [ ] I can simplify square roots like $\sqrt{72}$.
- [ ] I can solve multi-step linear equations and check my answer.
- [ ] I can solve inequalities and graph them on a number line.
- [ ] I can solve a system of two equations by substitution and by elimination.

Functions and Graphs

- [ ] I can explain what a function is and use $f(x)$ notation.
- [ ] I can find the domain and range from an equation, a graph, or a table.
- [ ] I can sketch the six parent functions from memory.
- [ ] I can find a slope and explain it as a rate with units.
- [ ] I can write the equation of a line from a graph, two points, or a word problem.
- [ ] I can find and explain an average rate of change.

Quadratics and Exponentials

- [ ] I can multiply polynomials and recognize the special products.
- [ ] I can factor using the GCF, trinomials, and the difference of squares.
- [ ] I can solve a quadratic by factoring, by square roots, and with the quadratic formula.
- [ ] I can find the vertex, zeros, and y-intercept of a parabola.
- [ ] I can write an exponential model from a percent increase or decrease.
- [ ] I can tell linear data from exponential data in a table.

Data and Statistics

- [ ] I can find the mean, median, quartiles, and IQR of a data set.
- [ ] I can identify outliers with the 1.5 × IQR rule.
- [ ] I can read and make dot plots, histograms, and box plots.
- [ ] I can interpret the slope and intercept of a line of fit in context.
- [ ] I can explain why correlation does not prove causation.

Habits and Communication

- [ ] I show every step, one line at a time.
- [ ] I check answers by substituting or estimating.
- [ ] I write answers in context, with units.
- [ ] I can explain my reasoning in complete sentences.
- [ ] I keep a mistakes notebook and review it.
- [ ] I can check my work with a graphing tool such as Desmos.

14 Practice

Answers and worked solutions are in section 15. Try everything first without looking, and show every step — use scratch paper for longer work.

A. Exponents and radicals

Simplify. Use only positive exponents.

1. $x^3 \cdot x^5$ _____
2. $(2a^2)^3$ _____
3. $\dfrac{12y^7}{3y^2}$ _____
4. $5^0 + 2^{-3}$ _____
5. $(3x^{-2}y)^2$ _____
6. $\sqrt{72}$ _____
7. $(4 \times 10^5)(3 \times 10^{-2})$ in scientific notation _____
8. $8^{2/3}$ **GET AHEAD** _____

B. Equations and inequalities

Solve. Graph the inequalities on a number line.

1. $3(x - 4) + 2 = 2x + 5$ _____
2. $\dfrac{2}{3}x - 5 = 7$ _____
3. $-4x + 7 \le 19$ _____
4. $|2x - 3| = 9$ _____
5. Solve $V = \pi r^2 h$ for h. _____
6. $2x + y = 7$ and $x - y = 2$ _____
7. $4x - 2(x + 1) = 2x + 3$ _____
8. $-5 < 2x + 1 \le 9$ _____

C. Lines and slope

1. Find the slope of the line through $(-2, 5)$ and $(4, -7)$. _____
2. Write the equation of that line in slope-intercept form. _____
3. Write the line parallel to $y = 3x - 1$ through $(2, 4)$. _____
4. Write the line perpendicular to $y = \frac{1}{2}x + 3$ through $(1, 1)$. _____
5. Find the intercepts and the slope of $3x + 4y = 12$. _____
6. A plumber charges \$65 plus \$40 per hour. Write $C(h)$, find the cost of a 3.5-hour job, and say what the slope means.

_____

D. Factor completely

1. $6x^2 + 9x$ _____
2. $x^2 + 7x + 12$ _____
3. $x^2 - 5x - 14$ _____
4. $16x^2 - 1$ _____
5. $2x^2 + 7x + 3$ _____
6. $3x^2 - 12$ _____
7. $x^3 + 2x^2 - 9x - 18$ _____

E. Quadratic equations

1. Solve $x^2 - 3x - 10 = 0$. _____
2. Solve $2x^2 = 50$. _____
3. Solve $(x - 3)^2 = 16$. _____
4. Solve $x^2 + 4x - 3 = 0$ with the quadratic formula. _____
5. How many real solutions does $3x^2 - 2x + 5 = 0$ have? How do you know? _____
6. Find the vertex and the axis of symmetry of $y = x^2 - 6x + 5$. _____
7. Solve $x^2 + 8x + 5 = 0$ by completing the square. **GET AHEAD** _____

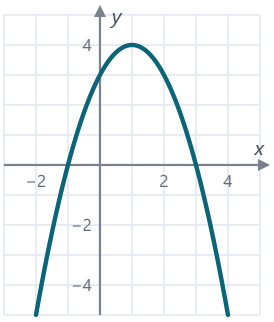
F. Exponential models

1. A town of 12,000 people grows 3% per year. Write $P(t)$ and find the population after 5 years. _____
2. A $20,000 car loses 15% of its value each year. Find its value after 4 years, to the nearest dollar. _____
3. $1,000 is invested at 4% interest compounded monthly. How much is there after 3 years? _____
4. 500 bacteria double every 4 hours. How many are there after one day? _____
5. Linear, exponential, or neither? Write an equation if you can.

x	0	1	2	3
(a)	5	15	45	135
(b)	5	8	11	14
(c)	1	2	4	7

G. Functions

1. For $f(x) = 2x^2 - 3x + 1$, find $f(-2)$ and $f(3)$. _____
2. Give the domain and range of $g(x) = \sqrt{x - 4}$. _____
3. Is {(1, 2), (2, 3), (1, 5)} a function? Explain. _____
4. Find the average rate of change of $f(x) = x^2$ from $x = 1$ to $x = 4$. _____
5. Compared with $y = x^2$, how is $g(x) = (x + 2)^2 - 5$ moved? What is its vertex? **GET AHEAD** _____
6. Use the graph at the right. Find (a) the vertex, (b) the zeros, (c) the y-intercept, (d) the range, (e) where the function is increasing, and (f) its equation in vertex form. **GET AHEAD** for (f).



H. Statistics

Ten students recorded how many minutes it takes them to get to school: **3, 5, 6, 6, 7, 8, 9, 10, 11, 25**

1. Find the mean and the median.

2. Find Q1, Q3, and the IQR. _____

3. Is 25 an outlier? Use the 1.5 × IQR rule.

4. Which measure of center better describes a typical student's trip? Why?

5. A study of 30 students found $\hat{y} = 62 + 4.5x$, where x = hours studied and $\hat{y}$ = predicted test score. Interpret the slope in context.

6. Predict the score for 6 hours of study. A student who studied 6 hours scored 85. What is the residual? _____

7. A bag holds 6 red, 3 blue, and 1 green marble. Find $P(\text{blue})$, $P(\text{not red})$, and the probability of drawing blue twice, with replacement.

I. AP-style free response calculator allowed

Two new apps launch on the same day. **App A** starts with 500 users and gains 50 new users each week. **App B** starts with 200 users, and its number of users grows by 8% each week.
(a) Write a function $A(t)$ for App A and a function $B(t)$ for App B, where t is the number of weeks since launch.
(b) Which app has more users after 12 weeks? Show the work that leads to your answer.
(c) Will App B ever have more users than App A? If so, find the first whole week when this happens. Justify your answer.

15 Answer Key

Each answer shows the key steps. For explanations, many wordings are correct; the models show one strong way to say it.

A. Exponents and radicals

1	x^8 (add exponents)
2	$8a^6$ ($2^3 = 8$; $2 \cdot 3 = 6$)
3	$4y^5$
4	$\frac{9}{8}$ (1 + ⅛)
5	$\frac{9y^2}{x^4}$
6	$6\sqrt{2}$ ($\sqrt{36} \cdot \sqrt{2}$)
7	1.2×10^4 (12×10^3)
8	4 ($(\sqrt[3]{8})^2 = 2^2$)

B. Equations and inequalities

1	$x = 15$ ($3x - 10 = 2x + 5$)
2	$x = 18$ ($\frac{2}{3}x = 12$)
3	$x \geq -3$ ($-4x \leq 12$; flip)
4	$x = 6$ **or** $x = -3$
5	$h = \frac{V}{\pi r^2}$
6	$(3, 1)$ (add: $3x = 9$)
7	**no solution** ($2x - 2 = 2x + 3 \rightarrow -2 = 3$)
8	$-3 < x \leq 4$ ($-6 < 2x \leq 8$)

C. Lines and slope

1	-2 ($-12 \div 6$)
2	$y = -2x + 1$
3	$y = 3x - 2$
4	$y = -2x + 3$ (slope -2)
5	**(4, 0) and (0, 3); slope $-\frac{3}{4}$**
6	$C(h) = 40h + 65$; **$205** — the cost rises $40 for each additional hour

D. Factor completely

1	$3x(2x + 3)$
2	$(x + 3)(x + 4)$
3	$(x - 7)(x + 2)$
4	$(4x + 1)(4x - 1)$
5	$(2x + 1)(x + 3)$ ($ac = 6 \rightarrow 6$ and 1)
6	$3(x + 2)(x - 2)$ (GCF first)
7	$(x + 2)(x + 3)(x - 3)$ ($x^2(x + 2) - 9(x + 2)$)

E. Quadratic equations

1	$x = 5$ **or** -2 (factor)
2	$x = \pm 5$
3	$x = 7$ **or** -1 ($x - 3 = \pm 4$)
4	$-2 \pm \sqrt{7} \approx 0.65$ or -4.65
5	**none** ($b^2 - 4ac = -56$)
6	**vertex (3, −4)**; $x = 3$
7	$-4 \pm \sqrt{11}$ ($(x + 4)^2 = 11$)

F. Exponential models

1	$P(t) = 12000(1.03)^t$; **≈ 13,911**
2	**$10,440** ($20000 \cdot 0.85^4$)
3	**$1,127.27** ($n = 12$, $t = 3$)
4	**32,000** ($500 \cdot 2^6$)
5	**(a) exponential,** $y = 5 \cdot 3^x$ **(b) linear,** $y = 3x + 5$ **(c) neither** (differences 1, 2, 3; ratios 2, 2, 1.75)

G. Functions

1	$f(-2) = 15$, $f(3) = 10$ (8 + 6 + 1; 18 − 9 + 1)
2	**domain $x \geq 4$; range $y \geq 0$**
3	**No** — the input 1 has two outputs, 2 and 5.
4	5 ($(16 - 1) \div (4 - 1)$)
5	**left 2, down 5; vertex (−2, −5)**
6	**(a) (1, 4) (b)** $x = -1$ **and** 3 **(c) (0, 3) (d)** $y \leq 4$ **(e)** $x < 1$ **(f)** $y = -(x - 1)^2 + 4$

H. Statistics

1	**mean 9** (90 ÷ 10)**; median 7.5** ((7 + 8) ÷ 2)
2	**Q1 = 6, Q3 = 10, IQR = 4**
3	**Yes** — 25 is above the fence 10 + 6 = 16.
4	**The median** — 25 pulls the mean up to 9.
5	**For each additional hour studied, the predicted score increases by 4.5 points.**
6	**89 points; residual** $85 - 89 = -4$
7	**3/10, 2/5, 9/100** (3/10 · 3/10)

I. Model free response

(a) $A(t) = 500 + 50t$ and $B(t) = 200(1.08)^t$, where t is weeks since launch and the outputs are numbers of users.
(b) $A(12) = 500 + 50(12) = 1{,}100$ users. $B(12) = 200(1.08)^{12} \approx 504$ users. After 12 weeks, App A has more users.
(c) Yes. App A always gains the same 50 users per week, but App B gains 8% of its current users, so its weekly gain keeps growing. Once App B has more than 625 users (because 8% of 625 is 50), it gains more each week than App A, and the gap keeps shrinking until App B passes App A. A table shows when: in week 29, $A(29) = 1{,}950$ and $B(29) \approx 1{,}863$, so App A is still ahead; in week 30, $A(30) = 2{,}000$ and $B(30) \approx 2{,}013$. App B first has more users in **week 30**.

Why it works: both functions are defined in context · part (b) shows the calculations, not just a verdict · part (c) gives a reason (a constant gain against a percent gain) and evidence on both sides of the crossing · each conclusion is a complete sentence with units.

Keep going!

There is no royal road to geometry — or to calculus. But there is a road, and every problem you finish this year is a step on it. Keep showing your work, keep checking it, and keep explaining what it means. Three years from now, the AP exam will feel like the next step, not a leap.